DEPARTMENT OF MATHEMATICS

SYLLABUS FOR Ph. D ADMISSION IN MATHEMATICS

Real Analysis and Calculus

Sequences and series of functions, uniform convergence- Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Metric spaces, compactness, connectedness.

Multivariable Calculus: Functions of several variables, Continuity and differentiability, Partial and directional derivatives, Total derivative, Maxima, minima, and saddle points, Lagrange multipliers. Multiple Integrals: Double and triple integrals, Applications to Area, Surface area and Volume.

Vector Calculus: gradient, divergence and curl, Line integrals and Surface integrals, Green's theorem, Stokes' theorem and Gauss divergence theorem.

Complex Analysis

Algebra of complex numbers, Analytic functions, Cauchy-Riemann Equations, transcendental functions such as exponential, trigonometric and hyperbolic, Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Number Theory and Algebra

Number Theory: Divisors and Divisors function, relatively primes and Euler's totient function, Congruences and Modular Arithmetic, Chinese remainder theorem, Fermat's, Euler's, and Wilson's theorems.

Algebra: Groups and subgroups, Normal subgroups, quotient groups, homomorphisms, cyclic and permutation groups, Sylow theorems and applications. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain, Polynomial rings, Eisenstein irreducibility criterion. Infinite and finite fields, field extensions.

Linear Algebra

Algebra of matrices: Row reduction and Echelon forms, rank and solutions of linear systems, Eigenvalues and eigenvectors, Cayley-Hamilton theorem, Minimal polynomial, Algebraic-geometric multiplicities, diagonalization. special matrices, Quadratic forms.

Vector spaces and subspaces, linear dependence and independence, bases, dimensions. Linear transformations, algebra of linear transformations, rank and nullity, matrix representation of linear transformations. Inner product spaces, orthonormal basis, Gram-Schmidt orthogonalization.

Ordinary and Partial Differential Equations

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function

Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Numerical Analysis

Numerical solutions of algebraic equations: Bisection, Regula-Falsi, Iteration and Newton-Raphson methods, Rate of convergence. Interpolation: Finite differences, equal and unequally spaced arguments. Numerical differentiation and integration, Numerical solutions of ODEs using Taylor, Euler, modified Euler and Runge-Kutta methods.

Probability and Statistics

Probability, Bayes theorem, random variables, moments, univariate distributions (binomial, Poisson, geometric, normal, gamma, exponential and Weibull); Joint distributions: marginal and conditional. Joint characteristic functions. Probability inequalities (Tchebysheff, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case).

Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes.

Sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range. Methods of estimation, properties of estimators, confidence intervals. Tests of hypotheses: Large and Small sample tests.

Linear Programming

Linear programming problem, convex sets, extreme points; Basic feasible solution, graphical method, simplex method, duality. Transportation and Assignment Problems. Elementary queuing and inventory models. Steady-state solutions of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1.